EXERCISES FOR SECTION 2.5

In Exercises 1–16, find dy/dx by implicit differentiation.

1. $x^2 + y^2 = 36$
2. $x^2 - y^2 = 16$
3. $x^{1/2} + y^{1/2} = 9$
4. $x^3 + y^3 = 8$
5. $x^3 - xy + y^2 = 4$
6. $x^2y + y^2x = -2$
7. $x^3y^3 - y = x$
8. $\sqrt{xy} = x - 2y$
9. $x^3 - 3x^2y + 2xy^2 = 12$
10. $2 \sin x \cos y = 1$
11. $\sin x + 2 \cos 2y = 1$
12. $(\sin \pi x + \cos \pi y)^2 = 2$
13. $\sin x = x(1 + \tan y)$
14. $\cot y = x - y$
15. $y = \sin(xy)$
16. $x = \sec \dfrac{1}{y}$

In Exercises 17–20, (a) find two explicit functions by solving the equation for y in terms of x, (b) sketch the graph of the equation and label the parts given by the corresponding explicit functions, (c) differentiate the explicit functions, and (d) find dy/dx implicitly and show that the result is equivalent to that of part (c).

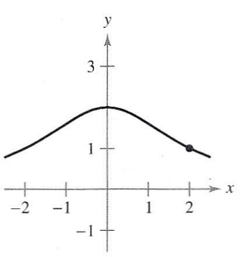
17. $x^2 + y^2 = 16$
18. $x^2 + y^2 - 4x + 6y + 9 = 0$
19. $9x^2 + 16y^2 = 144$
20. $9y^2 - x^2 = 9$

In Exercises 21–28, find dy/dx by implicit differentiation and evaluate the derivative at the indicated point.

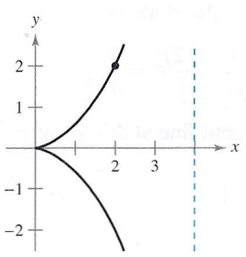
Equation	*Point*
21. $xy = 4$	$(-4, -1)$
22. $x^2 - y^3 = 0$	$(1, 1)$
23. $y^2 = \dfrac{x^2 - 4}{x^2 + 4}$	$(2, 0)$
24. $(x + y)^3 = x^3 + y^3$	$(-1, 1)$
25. $x^{2/3} + y^{2/3} = 5$	$(8, 1)$
26. $x^3 + y^3 = 4xy + 1$	$(2, 1)$
27. $\tan(x + y) = x$	$(0, 0)$
28. $x \cos y = 1$	$\left(2, \dfrac{\pi}{3}\right)$

In Exercises 29–32, find the slope of the tangent line to the graph at the indicated point.

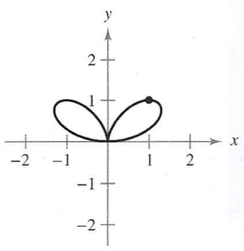
29. Witch of Agnesi:
$(x^2 + 4)y = 8$
Point: $(2, 1)$



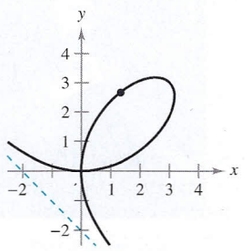
30. Cissoid:
$(4 - x)y^2 = x^3$
Point: $(2, 2)$



31. Bifolium:
$(x^2 + y^2)^2 = 4x^2y$
Point: $(1, 1)$



32. Folium of Descartes:
$x^3 + y^3 - 6xy = 0$
Point: $\left(\frac{4}{3}, \frac{8}{3}\right)$



In Exercises 33 and 34, find dy/dx implicitly and find the largest interval of the form $-a < y < a$ such that y is a differentiable function of x. Then express dy/dx as a function of x.

33. $\tan y = x$
34. $\cos y = x$

In Exercises 35–40, find d^2y/dx^2 in terms of x and y.

35. $x^2 + y^2 = 36$
36. $x^2y^2 - 2x = 3$
37. $x^2 - y^2 = 16$
38. $1 - xy = x - y$
39. $y^2 = x^3$
40. $y^2 = 4x$

In Exercises 41 and 42, use a graphing utility to graph the equation. Find an equation of the tangent line to the graph at the indicated point and sketch its graph.

41. $\sqrt{x} + \sqrt{y} = 4, \quad (9, 1)$
42. $y^2 = \dfrac{x - 1}{x^2 + 1}, \quad \left(2, \dfrac{\sqrt{5}}{5}\right)$

In Exercises 43 and 44, find equations for the tangent line and normal line to the circle at the indicated points. (The *normal line* at a point is perpendicular to the tangent line at the point.) Use a graphing utility to graph the equation, tangent line, and normal line.

43. $x^2 + y^2 = 25$
$(4, 3), (-3, 4)$
44. $x^2 + y^2 = 9$
$(0, 3), (2, \sqrt{5})$

45. Show that the normal line at any point on the circle $x^2 + y^2 = r^2$ passes through the origin.
46. Two circles of radius 4 are tangent to the graph of $y^2 = 4x$ at the point $(1, 2)$. Find equations of these two circles.

In Exercises 47 and 48, find the points at which the graph of the equation has a vertical or horizontal tangent line.

47. $25x^2 + 16y^2 + 200x - 160y + 400 = 0$
48. $4x^2 + y^2 - 8x + 4y + 4 = 0$